

Landauer-Büttiker-type current formula for hybrid mesoscopic systems

Z. Y. Zeng^{1,2}, F. Claro² and Baowen Li¹

¹ *Department of Physics, National University of Singapore, 10 Kent Ridge Crescent, 119260 Singapore*

² *Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile*

(Dated: February 1, 2008)

A general Landauer-Büttiker-type current formula is derived, which can be applied to the ferromagnet (**F**)/Normal metal (**N**)/superconductor (**S**), **F/N/N**, **N/N/S** and **N/N/N** systems, even in the presence of interactions in the central region.

PACS numbers: 72.10.Bg, 72.25.-b, 74.25.Fy, 73.40.-c

I. INTRODUCTION

Electronic transport in mesoscopic systems or nanoscopic structures has received extensive and intensive theoretical and experimental attention¹. In mesoscopic systems the sample size is smaller than the phase coherent length, electrons retain their phase when travelling through the sample. In the ballistic limit, i.e., when the dimensions of the sample is smaller than the mean free path, electrons can traverse the system without any scattering. In contrast to macroscopic systems, the conductance of mesoscopic systems is sample-specific, since electron wavefunctions are strongly dependent on the form of the boundary of the sample and the configuration of scatterers located within the sample. To calculate the conductance of mesoscopic samples, one should at first consider the wave nature of electrons. The Boltzmann transport equation² is obviously inappropriate, since the preassumption that electrons can be viewed as classical particles does not hold at a mesoscopic scale due to the Heisenberg uncertainty limitation. So one should resort to other theoretical approaches such as linear response theory.³ However, in fact, electronic transport in solids is equivalent to the wave transmission of electrons through a generalized potential barrier, which can be associated with a scattering matrix. In measuring the conductance of a sample, one always connects the sample to some contacts through perfect leads⁴. In a two-terminal setup (L=left, R=right), the Landauer-Büttiker formula⁵ states that the current \mathcal{I} can be expressed as a convolution of the transmission probability \mathcal{T} and the Fermi distribution function f_α ($\alpha = L, R$), i.e., $\mathcal{I} = \frac{2e}{h} \int \mathcal{T}(\epsilon) [f_L(\epsilon) - f_R(\epsilon)] d\epsilon$. The conductance \mathcal{G} in the linear response regime is $\mathcal{G} = \frac{2e^2}{h} \int \mathcal{T}(\epsilon) (-\frac{\partial f}{\partial \epsilon}) d\epsilon$. Such a formulation seems more appealing since the transport properties are encoded in the corresponding transmission probability, which can be calculated by various methods.

Due to the recent development in nanofabrication and material growth technologies, several kinds of mesoscopic hybrid structures have been realized experimentally. These nanoscale structures include normal-metal/superconductor nanostructures (**N/S**)⁶, superconductor-insulator/superconductor junctions (**S/I/S**)⁸, superconductor/quantum-dot/superconductor

transistors (**S/QD/S**)⁹, normal metal/superconducting quantum-dot/normal metal transistors (**N/SQD/N**)¹⁰, ferromagnet/superconductor (**F/S**) contact¹¹, superconductor/ferromagnet/superconductor sandwich structure (**S/F/S**)¹² and normal-metal/ferromagnetic-quantum-dot/normal metal (**N/FQD/N**) transistors¹³. In the presence of a superconductor component, Andreev reflection¹⁴ dominates the transport process in the low bias case. The imbalance between the spin-up and spin-down density of states at the Fermi level for ferromagnetic materials introduces the spin-dependent transport¹⁵. The conductance of a **F/S** junction is predicted to be smaller or larger than the **N/S** case, depending on the ratio between the spin-up and spin-down density of states¹⁶.

The nonequilibrium Green's function (NEGF) approach¹⁷ has proven to be a powerful technique to investigate the transport problem in mesoscopic systems. Using the NEGF method, Meir and Wingreen¹⁸ have derived a Landauer-Büttiker-type formula for transport through an interacting normal metal attached to two normal leads. Later, some groups have applied the NEGF method to the **N/NQD/S**¹⁹ and **S/NQD/S**²⁰ cases. In this paper, we extend the NEGF theory to a mesoscopic hybrid **F/N/S** structure, obtaining a Landauer-Büttiker-type current formula, which allows us to investigate the spin-dependent current and Andreev reflecting current in a unified way.

II. FORMULATION OF THE PROBLEM AND CURRENT FORMULAS

We consider electron tunneling through a ferromagnet/normal metal(semiconductor)/superconductor hybrid structure. Under the mean-field approximation, the ferromagnet is characterized by a molecular magnetic moment **M**, making at an angle θ with the **F/N** interface²¹, while the BCS Hamiltonian is adopted for the superconductor, with an order parameter Δ describing its energy gap²². In the central region which contains a normal metal, we take into consideration various kinds of couplings, such as the electron-electron interaction, electron-phonon interaction, etc. Then the hamiltonian

can be written as

$$H = H_F + H_S + H_D + H_T, \quad (1)$$

in which

$$H_F = \sum_{k\sigma} (\epsilon_{k\sigma} + \sigma M \cos \theta) f_{k\sigma}^\dagger f_{k\sigma} + \sum_{k\sigma} M \sin \theta f_{k\sigma}^\dagger f_{k\sigma}^-, \quad (2)$$

$$H_S = \sum_{p\sigma} \epsilon_{p\sigma} s_{p\sigma}^\dagger s_{p\sigma} + \sum_p \left[\Delta^* s_{p\uparrow}^\dagger s_{-p\downarrow}^\dagger + \Delta s_{p\uparrow} s_{-p\downarrow} \right] \quad (3)$$

$$H_C = \sum_{n\sigma} \epsilon_{n\sigma} c_{n\sigma}^\dagger c_{n\sigma} + H_{int}(\{c_{n\sigma}^\dagger\}, \{c_{n\sigma}\}) \quad (4)$$

$$H_T = \sum_{kn;\sigma} \left[T_{kn;\sigma}^L f_{k\sigma}^\dagger c_{n\sigma} + T_{kn;\sigma}^{L*} c_{n\sigma}^\dagger f_{k\sigma} \right] + \sum_{pn;\sigma} \left[T_{pn;\sigma}^R s_{p\sigma}^\dagger c_{n\sigma} + T_{pn;\sigma}^{R*} c_{n\sigma}^\dagger s_{p\sigma} \right], \quad (5)$$

are the hamiltonian for ferromagnet, superconductor, central normal metal and tunneling hamiltonian, respectively. In Eqs. (2-5), $f(f^\dagger)$, $s(s^\dagger)$ and $c(c^\dagger)$ represent the electron annihilation (creation) operator of the ferromagnet, superconductor and normal metal, respectively; $T^{L/R}$ denotes the tunneling matrix between the normal metal and the left ferromagnet (right superconductor), H_{int} is the interaction term in the central normal metal, which allows various kinds of interaction to be included. Throughout this paper, we assume Δ is real for convenience.

The current flowing into the central region from the left ferromagnet can be evaluated from the time evolution of the total electron number operator in the left lead¹⁷

$$J_L = -e \left\langle \frac{dN_L(t)}{dt} \right\rangle = \frac{ie}{\hbar} \langle [N_L, H] \rangle$$

$$\begin{aligned} &= \frac{ie}{\hbar} \sum_{kn,\sigma} (T_{kn;\sigma}^L \langle f_{k\sigma}^\dagger(t) c_{n\sigma}(t) \rangle - T_{kn;\sigma}^{L*} \langle c_{n\sigma}^\dagger(t) f_{k\sigma}(t) \rangle) \\ &= -\frac{e}{\hbar} Re \sum_{kn}^{i=1,3} \hat{\mathbf{T}}_{kn;ii}^{L\dagger} \hat{\mathbf{G}}_{kn;ii}^<(t, t). \end{aligned} \quad (6)$$

Here we have expressed various kinds of Green's functions in a generalized Nambu representation, spanned in the spin-dependent particle-hole space, in which the spin effect and Andreev reflection are considered on the same footing and treated in a unified way. The Green's functions in the generalized Nambu space are of the form

$$\begin{aligned} \hat{\mathbf{G}}_{XY}^{</>}(t, t') &= \sum_{ij} \hat{\mathbf{G}}_{XiYj}^{</>}(t, t') \\ &= \pm i \sum_{ij} \langle \mathbf{Y}_j^\dagger(t') / \mathbf{X}_i(t) \otimes \mathbf{X}_i(t) / \mathbf{Y}_j^\dagger(t') \rangle, \end{aligned} \quad (7)$$

$$\begin{aligned} \hat{\mathbf{G}}_{XY}^{r/a}(t, t') &= \sum_{ij} \hat{\mathbf{G}}_{XiYj}^{r/a}(t, t') \\ &= \mp i \sum_{ij} \theta(\pm t \mp t') \langle [\mathbf{X}_i(t) \otimes \mathbf{Y}_j^\dagger(t') - \mathbf{Y}_j^\dagger(t') \otimes \mathbf{X}_i(t)] \rangle, \end{aligned} \quad (8)$$

in which the four component of the spin-dependent particle-hole vector reads

$$\mathbf{X}_i = (X_{i\uparrow} \quad X_{i\downarrow}^\dagger \quad X_{i\downarrow} \quad X_{i\uparrow}^\dagger)^\dagger. \quad (9)$$

The tunneling matrix takes the form in the generalized Nambu space

$$\hat{\mathbf{T}}_{kn}^L / \hat{\mathbf{T}}_{pn}^R = \begin{pmatrix} T_{kn;\uparrow}^L / T_{pn;\uparrow}^R & 0 & 0 & 0 \\ 0 & -T_{kn;\downarrow}^{L*} / T_{pn;\downarrow}^{R*} & 0 & 0 \\ 0 & 0 & T_{kn;\downarrow}^L / T_{pn;\downarrow}^R & 0 \\ 0 & 0 & 0 & -T_{kn;\uparrow}^{L*} / T_{pn;\uparrow}^{R*} \end{pmatrix}. \quad (10)$$

With the help of Dyson's equation, $\hat{\mathbf{G}}_{kn}^<$ can be decoupled into the product of the unperturbed Green's function $\hat{\mathbf{g}}_{kk}$ of the ferromagnet and the Green's function $\hat{\mathbf{G}}_{mn}$ of the central normal metal regime in the presence of elastic coupling to the outside world

$$\begin{aligned} \hat{\mathbf{G}}_{kn}^<(t, t') &= \sum_m \int dt_1 [\hat{\mathbf{g}}_{kk}^<(t, t_1) \hat{\mathbf{T}}_{km}^L \hat{\mathbf{G}}_{mn}^r(t_1, t') + \hat{\mathbf{g}}_{kk}^r(t, t_1) \\ &\quad \hat{\mathbf{T}}_{km}^L \hat{\mathbf{G}}_{mn}^<(t_1, t')]. \end{aligned} \quad (11)$$

For convenience we introduce the linewidth matrices in

the generalized Nambu representation

$$\hat{\Gamma}^L(\omega) = 2\pi \hat{\mathbf{T}}_{kn}^{L\dagger} \hat{\rho}_L(\omega) \hat{\mathbf{T}}_{kn}^L = i[\hat{\Sigma}_L^r(\omega) - \hat{\Sigma}_L^a(\omega)] \quad (12)$$

$$\hat{\Gamma}^R(\omega) = 2\pi \hat{\mathbf{T}}_{pn}^{R\dagger} \hat{\rho}_R(\omega) \hat{\mathbf{T}}_{pn}^R = i[\hat{\Sigma}_R^r(\omega) - \hat{\Sigma}_R^a(\omega)] \quad (13)$$

where (see Appendix)

$$\hat{\Sigma}_L^{r/a}(\omega) = \hat{\mathbf{T}}_{kn}^{L\dagger} \hat{\mathbf{g}}_{ff}^{r/a}(\omega) \hat{\mathbf{T}}_{kn}^L \quad (14)$$

$$\begin{aligned}
&= \mp \frac{i}{2} \begin{pmatrix} \Gamma_{L+} & 0 & \Gamma_{L-} & 0 \\ 0 & \Gamma'_{L+} & 0 & \Gamma_{L-} \\ \Gamma_{L-} & 0 & \Gamma'_{L+} & 0 \\ 0 & \Gamma_{L-} & 0 & \Gamma_{L+} \end{pmatrix}, \\
\hat{\Sigma}_R^{r/a}(\omega) &= \hat{\mathbf{T}}_{pn}^{R\dagger} \hat{\mathbf{g}}_{ss}^{r/a}(\omega) \hat{\mathbf{T}}_{pm}^R \quad (15) \\
&= \mp \frac{i}{2} \begin{pmatrix} \Gamma_{RD} & -\Gamma_{RND} & 0 & 0 \\ -\Gamma_{RND} & \Gamma_{RD} & 0 & 0 \\ 0 & 0 & \Gamma_{RD} & \Gamma_{RND} \\ 0 & 0 & \Gamma_{RND} & \Gamma_{RD} \end{pmatrix},
\end{aligned}$$

in which

$$\Gamma_{L+} = \cos^2 \frac{\theta}{2} \Gamma_{L\uparrow} + \sin^2 \frac{\theta}{2} \Gamma_{L\downarrow}, \quad (16)$$

$$\Gamma'_{L+} = \cos^2 \frac{\theta}{2} \Gamma_{L\downarrow} + \sin^2 \frac{\theta}{2} \Gamma_{L\uparrow}, \quad (17)$$

$$\Gamma_{L-} = \frac{\sin \theta}{2} (\Gamma_{L\uparrow} - \Gamma_{L\downarrow}), \quad (18)$$

$$\Gamma_{RD} = \rho_S(\omega) \Gamma_R, \quad (19)$$

$$\Gamma_{RND} = \rho_S(\omega) \Gamma_R \frac{\Delta}{\omega}, \quad (20)$$

with

$$\Gamma_{L\uparrow} = 2\pi \rho_{L\uparrow} T_{kn}^{L*} T_{km}^L, \quad (21)$$

$$\Gamma_{L\downarrow} = 2\pi \rho_{L\downarrow} T_{kn}^{L*} T_{km}^L, \quad (22)$$

$$\Gamma_R = 2\pi \rho_S^N T_{pn}^{R*} T_{pm}^R, \quad (23)$$

$$\rho_S(\omega) = \frac{|\omega| \theta(|\omega| - \Delta)}{\sqrt{\omega^2 - \Delta^2}}. \quad (24)$$

In the above equations, $\rho_{L\uparrow}$ and $\rho_{L\downarrow}$ are the spin-up and spin-down density of states in the ferromagnetic region, respectively, ρ_S^N is the normal density of states when the superconductor order parameter $\Delta = 0$, while ρ_S is the dimensionless BCS density of states of the superconductor.

Substituting Eq. (11) into (6), we get the following compact form for the current

$$J_L = \frac{ie}{h} \sum_{i=1,3} \int d\omega \{ [\hat{\mathbf{G}}_{cc}^< + \hat{\mathbf{f}}_L (\hat{\mathbf{G}}_{cc}^r - \hat{\mathbf{G}}_{cc}^a)] \hat{\mathbf{\Gamma}}^L \}_{ii}. \quad (25)$$

In deriving Eq. (25), we have used the fluctuation-dissipation theorem $\hat{\mathbf{g}}^< = \hat{\mathbf{f}}(\hat{\mathbf{g}}^a - \hat{\mathbf{g}}^r)$, where $\hat{\mathbf{f}}$ is the Fermi distribution function matrix in the generalized Nambu representation, given by

$$\hat{\mathbf{f}}_{L/R} = (\hat{f}_{L/R;ij}). \quad (26)$$

Here $\hat{f}_{L/R;ij}(x) = \delta_{ij} f_{L/R}(\omega + (-1)^i \mu_{L/R})$, with $\mu_{L/R}$ the chemical potential of the left ferromagnet/right superconductor. Similarly, one can derive the current flows into the right superconductor

$$J_R = \frac{ie}{h} \sum_{i=1,3} \int d\omega \{ [\hat{\mathbf{G}}_{cc}^< + \hat{\mathbf{f}}_R (\hat{\mathbf{G}}_{cc}^r - \hat{\mathbf{G}}_{cc}^a)] \hat{\mathbf{\Gamma}}^R \}_{ii}. \quad (27)$$

Combining the Eqs. (26) and (27), we get the following current formula($\hat{J} = (\hat{J}_L - \hat{J}_R)/2$)

$$\begin{aligned}
J &= \frac{ie}{2h} \sum_{i=1,3} \int d\omega \{ (\hat{\mathbf{\Gamma}}^L \hat{\mathbf{f}}_L - \hat{\mathbf{\Gamma}}^R \hat{\mathbf{f}}_R) (\hat{\mathbf{G}}_{cc}^r - \hat{\mathbf{G}}_{cc}^a) + \\
&\quad (\hat{\mathbf{\Gamma}}^L - \hat{\mathbf{\Gamma}}^R) \hat{\mathbf{G}}_{cc}^< \}_{ii}, \quad (28)
\end{aligned}$$

which is the central result of this work. It is the analogy of Meir-wingreen¹⁸ formula in the presence of both ferromagnetic and superconducting leads, providing a framework to study transport in mesoscopic hybrid interacting structures.

In the absence of interactions within the central region, one has the following Keldysh equation

$$\hat{\mathbf{G}}_{cc}^< = \hat{\mathbf{G}}_{cc}^r \hat{\Sigma}^< \hat{\mathbf{G}}_{cc}^a, \quad (29)$$

$$\hat{\Sigma}^< = \hat{\Sigma}_L^< + \hat{\Sigma}_R^< = i(\hat{\mathbf{f}}_L \hat{\mathbf{\Gamma}}^L + \hat{\mathbf{f}}_R \hat{\mathbf{\Gamma}}^R), \quad (30)$$

and the equality

$$\hat{\mathbf{G}}_{cc}^r - \hat{\mathbf{G}}_{cc}^a = -i \hat{\mathbf{G}}_{cc}^r \hat{\mathbf{\Gamma}} \hat{\mathbf{G}}_{cc}^a, \quad (31)$$

$$\hat{\mathbf{\Gamma}} = \hat{\mathbf{\Gamma}}^L + \hat{\mathbf{\Gamma}}^R. \quad (32)$$

With the help of Eqs. (29-32), Eq. (28) can be simplified into the following Landauer-Büttiker-type current formula for the noninteracting **F/N/S** mesoscopic hybrid structure (setting $\mu_R = 0$, $\mu_L = eV$ due to gauge invariance)

$$J = J^{NC} + J^A, \quad (33)$$

in which

$$\begin{aligned}
J^{NC} &= \frac{e}{h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] \sum_{i=1,3} \\
&\quad [\hat{\mathbf{G}}_{cc}^r \hat{\mathbf{\Gamma}}^R \hat{\mathbf{G}}_{cc}^a \hat{\mathbf{\Gamma}}^L]_{ii}, \quad (34)
\end{aligned}$$

$$\begin{aligned}
J^A &= \frac{e}{h} \int d\omega [f_L(\omega - eV) - f_L(\omega + eV)] \sum_{i=1,3}^{j=2,4} \hat{G}_{cc;ij}^r \\
&\quad (\hat{\mathbf{\Gamma}}^L \hat{\mathbf{G}}_{cc}^a \hat{\mathbf{\Gamma}}^L)_{ji}. \quad (35)
\end{aligned}$$

From the expression (33) for the current, one sees that the current comes from different physical process: (1) J^{NC} includes (a) conventional electron tunneling current, (b) tunneling current of an electron in the ferromagnet into the superconductor either as a hole or as a cooper-pair by picking up another electron; (2) J^A is the Andreev reflection current, representing an incident electron from the ferromagnet is reflected as a hole backwards into the ferromagnet, with a cooper-pair left in the superconductor. At zero temperature, when $eV < \Delta$, J^{NC} becomes zero (ρ_S in $\hat{\mathbf{\Gamma}}^R$ is zero when $eV < \Delta$) and the Andreev reflection dominates the current. Obviously, the current has a strong dependence on the polarization $\mathcal{P} = \frac{\rho_{L\uparrow} - \rho_{L\downarrow}}{\rho_{L\uparrow} + \rho_{L\downarrow}}$ of the ferromagnet and the direction θ of the magnetic moment \mathbf{M} , through the coupling matrix $\hat{\mathbf{\Gamma}}^L$ (notice that the Green's functions of the central region is also implicitly dependent on $\hat{\mathbf{\Gamma}}^L$). Equation

(28) and Equations (33-35) can be applied to the interacting and noninteracting mesoscopic **F/N/S**, **N/N/S**, **F/N/N** and **N/N/N** structures, respectively.

III. EXAMPLES

In order to illustrate how formulas (33-35) are applied to the **F/N/S**, **N/N/S**, **F/N/N** and **N/N/N** structures,

we discuss in what follows a simple case, in which the interaction in the central region is omitted, and only one level is relevant to the transport,

Consider the hamiltonian of the central region, $H_C = \epsilon_c c^\dagger c$. The unperturbed retarded/advanced Green's function $\hat{\mathbf{g}}_{cc}^{r/a}$ in the 4×4 spin-dependent particle-hole space is then

$$(\hat{\mathbf{g}}_{cc}^{r/a})^{-1} = \begin{pmatrix} \omega - \epsilon_c \pm i0^+ & 0 & 0 & 0 \\ 0 & \omega + \epsilon_c \pm i0^+ & 0 & 0 \\ 0 & 0 & \omega - \epsilon_c \pm i0^+ & 0 \\ 0 & 0 & 0 & \omega + \epsilon_c \pm i0^+ \end{pmatrix}. \quad (36)$$

From Dyson's equation, $\hat{\mathbf{G}}^{r/a} = \hat{\mathbf{g}}^{r/a} + \hat{\mathbf{g}}^{r/a} \hat{\Sigma}^{r/a} \hat{\mathbf{G}}^{r/a}$, one has for $\hat{\mathbf{G}}_{cc}^{r/a}$ the expression

$$\begin{aligned} \hat{\mathbf{G}}_{cc}^{r/a} &= (\text{adj}(\hat{\mathbf{A}})_{ij}) \det(\hat{\mathbf{A}})^{-1}, \\ \hat{\mathbf{A}} &= [(\hat{\mathbf{g}}_{cc}^{r/a})^{-1} - \hat{\Sigma}^{r/a}]^{-1}, \end{aligned} \quad (37)$$

where $\text{adj}(X)$ denotes the adjoint of X . Note that $\hat{\mathbf{G}}_{cc}^{r/a}$ is symmetric, i.e., $\hat{\mathbf{G}}_{cc;ij}^{r/a} = \hat{\mathbf{G}}_{cc;ji}^{r/a}$, as seen from its definition or Eq. (36).

$\hat{\Gamma}^L$ is diagonal, and $\hat{\mathbf{G}}_{cc}^{r/a}$ becomes

$$\hat{\mathbf{G}}_{cc}^{r/a} = \begin{pmatrix} \hat{\mathbf{G}}_1^{r/a} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}} & \hat{\mathbf{G}}_2^{r/a} \end{pmatrix}, \quad (38)$$

A. current through a F/N/S structure

For simplicity, we just consider here $\theta = 0$. Then the self-energy matrix $\hat{\Sigma}_L^{r/a}$ as well as the linewidth matrix

where

$$\hat{\mathbf{G}}_1^{r/a} = B_1^{-1} \begin{pmatrix} \omega + \epsilon_c \pm i(\Gamma_{L\downarrow} + \Gamma_{RD}) & \mp i\Gamma_{RND} \\ \mp i\Gamma_{RND} & \omega - \epsilon_c \pm i(\Gamma_{L\uparrow} + \Gamma_{RD}) \end{pmatrix} \quad (39)$$

$$\hat{\mathbf{G}}_2^{r/a} = B_2^{-1} \begin{pmatrix} \omega + \epsilon_c \pm i(\Gamma_{L\uparrow} + \Gamma_{RD}) & \pm i\Gamma_{RND} \\ \pm i\Gamma_{RND} & \omega - \epsilon_c \pm i(\Gamma_{L\downarrow} + \Gamma_{RD}) \end{pmatrix} \quad (40)$$

with

$$\begin{aligned} B_1 &= \det(\hat{\mathbf{G}}_1^{r/a}), \\ B_2 &= \det(\hat{\mathbf{G}}_2^{r/a}). \end{aligned}$$

Then the current tunneling through the **F/N/S** structure is

$$J^{NC} = \frac{e\Gamma_{R\rho S}}{h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] \left\{ \Gamma_{L\uparrow} \left[\sum_{i=1,2} |\hat{G}_{cc;1i}^r|^2 - 2\frac{\Delta}{\omega} \text{Re}(\hat{G}_{cc;11}^r \hat{G}_{cc;12}^{r*}) \right] + \right.$$

$$\Gamma_{L\downarrow} \left[\sum_{i=3,4} |\hat{G}_{cc;3i}^r|^2 + 2 \frac{\Delta}{\omega} \text{Re}(G_{cc;33}^r G_{cc;34}^{r*}) \right] \Big\}, \quad (41)$$

$$J^A = \frac{e\Gamma_{L\uparrow}\Gamma_{L\downarrow}}{h} \int d\omega [f_L(\omega - eV) - f_L(\omega + eV)] [|\hat{G}_{cc;12}^r|^2 + |\hat{G}_{cc;34}^r|^2]. \quad (42)$$

The physical implications of Equations (41) and (42) are apparent. J^{NC} is directly proportional to the spin-dependent tunneling matrix Γ_σ due to the coupling between the central region and the ferromagnet, and Γ_R as well as ρ_S for the superconductor. The Andreev current is proportional to the spin-up and spin-down tunneling matrix $\Gamma_{L\uparrow}$ and $\Gamma_{L\downarrow}$, which means that an up(down)-spin electron(hole) incident from the ferromagnet will be reflected in the interface of **N/S** and re-enter the ferromagnet as a down(up)-spin hole(electron). In contrast to the **N/N/S** structure (below), the Andreev current is strongly dependent on the polarization of the ferromagnet $\mathcal{P} = \frac{\rho_{L\uparrow} - \rho_{L\downarrow}}{\rho_{L\uparrow} + \rho_{L\downarrow}}$ via the factor $\Gamma_{L\uparrow}\Gamma_{L\downarrow}$. When the ferromagnet is fully polarized, i.e., $\mathcal{P} = 1$ or $\mathcal{P} = -1$, the Andreev current will be zero since no state is available for the reflected particle with reverse spin.

B. current through a N/N/S structure

When the molecule magnetic moment \mathbf{M} is set to zero, the ferromagnet becomes a normal metal structure. The spin-up and spin-down density of states $\rho_{L\uparrow}$ and $\rho_{L\downarrow}$ will be the same. Then $\hat{\Gamma}_{L\uparrow} = \hat{\Gamma}_{L\downarrow} = \hat{\Gamma}_L$, and $\hat{G}_{cc;11}^r = \hat{G}_{cc;33}^r$, $\hat{G}_{cc;22}^r = \hat{G}_{cc;44}^r$, and $\hat{G}_{cc;12}^r = \hat{G}_{cc;21}^r = -\hat{G}_{cc;34}^r = -\hat{G}_{cc;43}^r$. Therefore the current through the **N/N/S** system turns into

$$\begin{aligned} J^{NC} &= \frac{2e\Gamma_R\Gamma_L\rho_S}{h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] \\ &\quad \left\{ \left[\sum_{i=1,2} |\hat{G}_{cc;1i}^r|^2 - 2 \frac{\Delta}{\omega} \text{Re}(\hat{G}_{cc;11}^r \hat{G}_{cc;12}^{r*}) \right] \right\}, \quad (43) \\ J^A &= \frac{2e\Gamma_L^2}{h} \int d\omega [f_L(\omega - eV) - f_L(\omega + eV)] \\ &\quad |\hat{G}_{cc;12}^r|^2, \quad (44) \end{aligned}$$

which is the same as the results for the **N/NQD/S** hybrid structure derived by Sun and co-workers¹⁹ and has been studied in detail.

C. current through a F/N/N structure

When the order parameter $\Delta = 0$, the superconductor can be viewed as a normal metal. In this case, one can observe that $\rho_S = 1$ and all the non-diagonal elements of the full Green's function $\hat{\mathbf{G}}_{cc}^{r/a}$ of the central region are zero. Then the Andreev tunneling current $J^A = 0$, and

J^{NC} is simplified as

$$J = J^{NC} = \frac{e\Gamma_R}{h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] [\Gamma_{L\uparrow} |\hat{G}_{cc;11}^r|^2 + \Gamma_{L\downarrow} |\hat{G}_{cc;33}^r|^2]. \quad (45)$$

Since, for normal metal, one can write $\Gamma_R = (\Gamma_{R\uparrow} + \Gamma_{R\downarrow})/2$, then one gets the following expression for the current

$$\begin{aligned} J &= J_{\uparrow\uparrow} + J_{\downarrow\downarrow} + J_{\uparrow\downarrow} + J_{\downarrow\uparrow}; \\ J_{\uparrow\uparrow} &= \frac{e\Gamma_{L\uparrow}\Gamma_{R\uparrow}}{2h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] |\hat{G}_{cc;11}^r|^2, \\ J_{\downarrow\downarrow} &= \frac{e\Gamma_{L\downarrow}\Gamma_{R\downarrow}}{2h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] |\hat{G}_{cc;33}^r|^2, \\ J_{\uparrow\downarrow} &= \frac{e\Gamma_{L\uparrow}\Gamma_{R\downarrow}}{2h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] |\hat{G}_{cc;11}^r|^2, \\ J_{\downarrow\uparrow} &= \frac{e\Gamma_{L\downarrow}\Gamma_{R\uparrow}}{2h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] \\ &\quad |\hat{G}_{cc;33}^r|^2. \quad (46) \end{aligned}$$

The physical meaning of Eq. (46) is very clear. The current into the right normal region comes from two kinds of physical process: one is that an electron exits from the ferromagnet and enters the central normal region, and without spin-flip it tunnels into the right normal metal; the other is the spin-flipped tunneling process, the up(down)-spin electron leaving the ferromagnet into the central normal metal, and after spin-flip, it gets into the right normal region with down(up) spin.

D. current through a N/N/N structure

Taking advantage of the results for an **F/N/N** structure, if one further assumes $\Gamma_{L\uparrow} = \Gamma_{L\downarrow}$, the **F/N/N** structure then turns into the **N/N/N** structure, and one easily recovers from Eq. (46) the well-known expression²³

$$\begin{aligned} J = J^{NC} &= \frac{2e\Gamma_L\Gamma_R}{h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] |\hat{G}_{cc;11}^r|^2 \\ &= \frac{2e}{h} \int d\omega [f_L(\omega - eV) - f_R(\omega)] \\ &\quad \frac{\Gamma_L\Gamma_R}{(\omega - \epsilon_c)^2 + (\Gamma_L + \Gamma_R)^2/4}. \quad (47) \end{aligned}$$

One sees from Eq. (47) that the transmission probability $T \propto \frac{\Gamma_L\Gamma_R}{(\omega - \epsilon_c)^2 + (\Gamma_L + \Gamma_R)^2/4}$ is of the Breit-Wigner form.

IV. CONCLUDING REMARKS

We have given a general Landauer-Büttiker current formula, which can be applied to the **F/N/S**, **N/N/S**, **F/N/N** and **N/N/N** structures, even in the presence of interactions within the central region. However, one finds that one usually can not divide the current into the spin-up and spin-down current parts. Then the polarization in the presence of a ferromagnetic and a superconducting component can not be defined by the current polarization. The reason is that the spin-up/down current can not be conserved during the transport process, due to the spin flip in the central region and Andreev reflection at the **N/S** interface. For example, a spin-up electron from the ferromagnet tunnels through the barrier into the central region, while another electron with spin-down will probably enter the superconductor. It is obvious in such case that the spin-up/down current is not conserved.

ACKNOWLEDGMENT

This work was supported by a {Singapore}, Cátedra Presidencial en Ciencias, FONDECYT 1990425, Chile and NSF grant No. 53112-0810 of Human Normal University, China (ZYZENG).

Appendix

In this Appendix we derive the retarded(advanced) self energy matrices $\hat{\Sigma}_L^{r/a}$ and $\hat{\Sigma}_L^{r/a}$, due to the coupling of electrons in the central normal metal to the left ferromagnet and the right superconductor. The key step is to get first the retarded(advanced) Green's function for the ferromagnet and the superconductor: $\hat{g}_{ff}^{r/a}$ and $\hat{g}_{ss}^{r/a}$. The most convenient way is to diagonalize the hamiltonian of the ferromagnet and superconductor. We first calculate the Green's function for the ferromagnet. Applying the following Bogoliubov-Valatin transformation²⁴ to the ferromagnet hamiltonian H_F

$$f_{k\sigma} = \cos(\theta/2)a_{k\sigma} - \sigma \sin(\theta/2)a_{k\bar{\sigma}}, \quad (48)$$

one has

$$H'_F = \sum_{k\sigma} (\epsilon_{k\sigma} + \sigma M) a_{k\sigma}^\dagger a_{k\sigma}. \quad (49)$$

Defining the following retarded(advanced) Green's function

$$\begin{aligned} g_{a\sigma;e}^{r/a}(t-t') &= \mp i\theta(\pm t \mp t') \sum_k \langle a_{k\sigma}(t), a_{k\sigma}^\dagger(t') \rangle \\ &= \mp i\theta(\pm t \mp t') \sum_k e^{-i(\epsilon_{k\sigma} + \sigma M)(t-t')/\hbar} \end{aligned} \quad (50)$$

$$\begin{aligned} g_{a\sigma;h}^{r/a}(t-t') &= \mp i\theta(\pm t \mp t') \sum_k \langle a_{k\sigma}(t), a_{k\sigma}^\dagger(t') \rangle \\ &= \mp i\theta(\pm t \mp t') \sum_k e^{i(\epsilon_{k\sigma} + \sigma M)(t-t')/\hbar} \end{aligned} \quad (51)$$

one finally gets for the ferromagnetic lead the retarded (advanced) Green's functions

$$\hat{g}_{ff}^{r/a}(t-t') = \sum_k \hat{g}_{ffk}^{r/a}(t-t') = (\hat{g}_{ff;mn}^{r/a}(t-t')), \quad (52)$$

where

$$\hat{g}_{ff;11}^{r/a}(t-t') = \cos^2 \frac{\theta}{2} g_{a\uparrow;e}^{r/a}(t-t') + \sin^2 \frac{\theta}{2} g_{a\downarrow;e}^{r/a}(t-t'), \quad (53)$$

$$\hat{g}_{ff;22}^{r/a}(t-t') = \cos^2 \frac{\theta}{2} g_{a\downarrow;h}^{r/a}(t-t') + \sin^2 \frac{\theta}{2} g_{a\uparrow;h}^{r/a}(t-t'), \quad (54)$$

$$\hat{g}_{ff;33}^{r/a}(t-t') = \cos^2 \frac{\theta}{2} g_{a\downarrow;e}^{r/a}(t-t') + \sin^2 \frac{\theta}{2} g_{a\uparrow;e}^{r/a}(t-t'), \quad (55)$$

$$\hat{g}_{ff;44}^{r/a}(t-t') = \cos^2 \frac{\theta}{2} g_{a\uparrow;h}^{r/a}(t-t') + \sin^2 \frac{\theta}{2} g_{a\downarrow;h}^{r/a}(t-t'), \quad (56)$$

$$\begin{aligned} \hat{g}_{ff;13}^{r/a}(t-t') &= \hat{g}_{ff;31}^{r/a}(t-t') \\ &= \frac{\sin \theta}{2} (g_{a\uparrow;e}^{r/a}(t-t') - g_{a\downarrow;e}^{r/a}(t-t')) \end{aligned} \quad (57)$$

$$\begin{aligned} \hat{g}_{ff;24}^{r/a}(t-t') &= \hat{g}_{ff;42}^{r/a}(t-t') \\ &= \frac{\sin \theta}{2} (g_{a\uparrow;h}^{r/a}(t-t') - g_{a\downarrow;h}^{r/a}(t-t')) \end{aligned} \quad (58)$$

$$\begin{aligned} \hat{g}_{ff;12}^{r/a}(t-t') &= \hat{g}_{ff;14}^{r/a}(t-t') = \hat{g}_{ff;21}^{r/a}(t-t') \\ &= \hat{g}_{ff;23}^{r/a}(t-t') = 0, \end{aligned} \quad (59)$$

$$\begin{aligned} \hat{g}_{ff;32}^{r/a}(t-t') &= \hat{g}_{ff;34}^{r/a}(t-t') = \hat{g}_{ff;41}^{r/a}(t-t') \\ &= \hat{g}_{ff;43}^{r/a}(t-t') = 0. \end{aligned} \quad (60)$$

The sum over k can be transformed into an integral, i.e., $\sum_k \rightarrow \int d\epsilon \rho_{L\sigma}$. Then one arrives at

$$\hat{g}_{ff;11}^{r/a}(t-t') = \mp i\delta(t-t') (\cos^2 \frac{\theta}{2} \rho_{L\uparrow} + \sin^2 \frac{\theta}{2} \rho_{L\downarrow}), \quad (61)$$

$$\hat{g}_{ff;22}^{r/a}(t-t') = \mp i\delta(t-t') (\cos^2 \frac{\theta}{2} \rho_{L\downarrow} + \sin^2 \frac{\theta}{2} \rho_{L\uparrow}), \quad (62)$$

$$\hat{g}_{ff;33}^{r/a}(t-t') = \mp i\delta(t-t') (\cos^2 \frac{\theta}{2} \rho_{L\downarrow} + \sin^2 \frac{\theta}{2} \rho_{L\uparrow}), \quad (63)$$

$$\hat{g}_{ff;44}^{r/a}(t-t') = \mp i\delta(t-t') (\cos^2 \frac{\theta}{2} \rho_{L\uparrow} + \sin^2 \frac{\theta}{2} \rho_{L\downarrow}), \quad (64)$$

$$\begin{aligned}
\hat{g}_{ff;13}^{r/a}(t-t') &= \hat{g}_{ff;31}^{r/a}(t-t') = \hat{g}_{ff;24}^{r/a}(t-t') \\
&= \hat{g}_{ff;42}^{r/a}(t-t') = \mp i\delta(t-t') \\
&\quad \frac{\sin\theta}{2}(\rho_{L\uparrow} - \rho_{L\downarrow}), \quad (65) \\
\hat{g}_{ff;12}^{r/a}(t-t') &= \hat{g}_{ff;14}^{r/a}(t-t') = \hat{g}_{ff;21}^{r/a}(t-t') \\
&= \hat{g}_{ff;23}^{r/a}(t-t') = 0, \quad (66) \\
\hat{g}_{ff;32}^{r/a}(t-t') &= \hat{g}_{ff;34}^{r/a}(t-t') = \hat{g}_{ff;41}^{r/a}(t-t') \\
&= \hat{g}_{ff;43}^{r/a}(t-t') = 0. \quad (67)
\end{aligned}$$

The matrix product $\hat{\mathbf{L}}_{kn}^\dagger \hat{\mathbf{g}}_{ff}^{r/a}(t-t') \hat{\mathbf{L}}_{km}$ after Fourier transformation $\int d\omega e^{i\omega t} F(t)$ yields the retarded(advanced) self-energy matrix $\hat{\Sigma}_L^{r/a}(\omega)$ Eq. (14).

The procedure to get the retarded(advanced) Green's functions for the superconductor lead is similar. After the following Bogoliubov-Valatin transformation

$$s_{p\uparrow} = \mu_p b_{p\uparrow} + \nu_p b_{-p\downarrow}^\dagger, \quad (68)$$

$$s_{-p\downarrow} = \mu_p b_{-p\downarrow} - \nu_p b_{p\uparrow}^\dagger, \quad (69)$$

with

$$\mu_p^2 = \frac{1}{2} \left(1 + \frac{\epsilon_{p\sigma}}{\epsilon_{p\sigma}^2 + \Delta^2} \right), \quad (70)$$

$$\nu_p^2 = \frac{1}{2} \left(1 - \frac{\epsilon_{p\sigma}}{\epsilon_{p\sigma}^2 + \Delta^2} \right), \quad (71)$$

the superconductor hamiltonian H_S is diagonalized in the following form

$$\begin{aligned}
H'_S &= \sum_p \sqrt{\epsilon_{p\sigma}^2 + \Delta^2} (b_{p\uparrow}^\dagger b_{p\uparrow} + b_{-p\downarrow}^\dagger b_{-p\downarrow}) + 2 \sum_{p\sigma} \epsilon_{p\sigma} \nu_p^2 - \\
&\quad 2\Delta \sum_p \mu_p \nu_p. \quad (72)
\end{aligned}$$

Using the notation

$$\begin{aligned}
g_{b\sigma;e}^{r/a}(t-t') &= \mp i\theta(\pm t \mp t') \sum_p \langle b_{p\sigma}(t), b_{p\sigma}^\dagger(t') \rangle \\
&= \mp i\theta(\pm t \mp t') \sum_p e^{-i\sqrt{\epsilon_{p\sigma}^2 + \Delta^2}(t-t')/\hbar} \quad (73) \\
g_{b\sigma;h}^{r/a}(t-t') &= \mp i\theta(\pm t \mp t') \sum_p \langle b_{p\sigma}(t), b_{p\sigma}^\dagger(t') \rangle \\
&= \mp i\theta(\pm t \mp t') \sum_k e^{i\sqrt{\epsilon_{p\sigma}^2 + \Delta^2}(t-t')/\hbar}, \quad (74)
\end{aligned}$$

one can express the retarded (advanced) Green's function matrix for the superconductor lead as

$$\hat{g}_{ss}^{r/a}(t-t') = \sum_p \hat{g}_{pp}^{r/a}(t-t') = (\hat{g}_{ss;mn}^{r/a}(t-t')), \quad (75)$$

where

$$\hat{g}_{ss;11}^{r/a}(t-t') = \mu_p^2 g_{b\uparrow;e}^{r/a}(t-t') + \nu_p^2 g_{b\downarrow;h}^{r/a}(t-t'), \quad (76)$$

$$\hat{g}_{ss;22}^{r/a}(t-t') = \mu_p^2 g_{b\downarrow;h}^{r/a}(t-t') + \nu_p^2 g_{b\uparrow;e}^{r/a}(t-t'), \quad (77)$$

$$\hat{g}_{ss;33}^{r/a}(t-t') = \mu_p^2 g_{b\downarrow;e}^{r/a}(t-t') + \nu_p^2 g_{b\uparrow;h}^{r/a}(t-t'), \quad (78)$$

$$\hat{g}_{ss;44}^{r/a}(t-t') = \mu_p^2 g_{b\uparrow;h}^{r/a}(t-t') + \nu_p^2 g_{b\downarrow;e}^{r/a}(t-t'), \quad (79)$$

$$\begin{aligned}
\hat{g}_{ss;12}^{r/a}(t-t') &= \hat{g}_{ss;21}^{r/a}(t-t') = \mu_p \nu_p (g_{b\downarrow;h}^{r/a}(t-t') \\
&\quad - g_{b\uparrow;e}^{r/a}(t-t')), \quad (80)
\end{aligned}$$

$$\begin{aligned}
\hat{g}_{ss;34}^{r/a}(t-t') &= \hat{g}_{ss;43}^{r/a}(t-t') = \mu_p \nu_p (g_{b\downarrow;e}^{r/a}(t-t') \\
&\quad - g_{b\uparrow;h}^{r/a}(t-t')), \quad (81)
\end{aligned}$$

$$\begin{aligned}
\hat{g}_{ss;13}^{r/a}(t-t') &= \hat{g}_{ss;14}^{r/a}(t-t') = \hat{g}_{ss;23}^{r/a}(t-t') \\
&= \hat{g}_{ss;24}^{r/a}(t-t') = 0, \quad (82)
\end{aligned}$$

$$\begin{aligned}
\hat{g}_{ss;31}^{r/a}(t-t') &= \hat{g}_{ss;32}^{r/a}(t-t') = \hat{g}_{ss;41}^{r/a}(t-t') \\
&= \hat{g}_{ss;42}^{r/a}(t-t') = 0. \quad (83)
\end{aligned}$$

Since

$$\begin{aligned}
&\int d\epsilon_p \left(\mu_p^2 e^{-i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar} + \nu_p^2 e^{i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar} \right) \\
&= \frac{1}{2} \int_{-\infty}^{\infty} d\epsilon_p (e^{-i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar} + \\
&\quad e^{i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar}) \\
&= \int_0^{\infty} d\epsilon \frac{\epsilon \theta(\epsilon - \Delta)}{\sqrt{\epsilon^2 - \Delta^2}} (e^{-i\epsilon\tau/\hbar} + e^{i\epsilon\tau/\hbar}) \\
&= \int_{-\infty}^{\infty} d\epsilon \frac{|\epsilon| \theta(|\epsilon| - \Delta)}{\sqrt{\epsilon^2 - \Delta^2}} e^{-i\epsilon\tau/\hbar}, \quad (84)
\end{aligned}$$

and

$$\begin{aligned}
&\int d\epsilon_p \mu_p \nu_p (e^{-i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar} - e^{i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar}) \\
&= \frac{1}{2} \int_{-\infty}^{\infty} d\epsilon_p \frac{\Delta}{\sqrt{\epsilon_p^2 + \Delta^2}} (e^{-i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar} - \\
&\quad e^{i\sqrt{\epsilon_p^2 + \Delta^2}\tau/\hbar}) \\
&= \int_0^{\infty} d\epsilon \frac{\Delta \theta(\epsilon - \Delta)}{\sqrt{\epsilon^2 - \Delta^2}} (e^{-i\epsilon\tau/\hbar} - e^{i\epsilon\tau/\hbar}) \\
&= - \int_{-\infty}^{\infty} d\epsilon \frac{|\epsilon| \theta(|\epsilon| - \Delta)}{\sqrt{\epsilon^2 - \Delta^2}} \frac{\Delta}{\epsilon} e^{-i\epsilon\tau/\hbar}, \quad (85)
\end{aligned}$$

then one obtains after changing the sum \sum_p into an integral $\int d\epsilon_p \rho_S^N$

$$\hat{g}_{ss;11}^{r/a}(t-t') = \hat{g}_{ss;22}^{r/a}(t-t') = \hat{g}_{ss;33}^{r/a}(t-t') = \hat{g}_{ss;44}^{r/a}(t-t') \quad (86)$$

$$= \mp i\theta(\pm t \mp t') \int d\epsilon \frac{|\epsilon|\theta(|\epsilon| - \Delta)}{\sqrt{\epsilon^2 - \Delta^2}} e^{-i\epsilon(t-t')/\hbar} \quad (87)$$

$$\hat{g}_{ss;12}^{r/a}(t-t') = \hat{g}_{ss;21}^{r/a}(t-t') = -\hat{g}_{ss;34}^{r/a}(t-t') = -\hat{g}_{ss;43}^{r/a}(t-t') \quad (88)$$

$$= \mp i\theta(\pm t \mp t') \int d\epsilon \frac{|\epsilon|\theta(|\epsilon| - \Delta)}{\sqrt{\epsilon^2 - \Delta^2}} \frac{\Delta}{\epsilon} e^{-i\epsilon(t-t')/\hbar}, \quad (89)$$

$$\hat{g}_{ss;13}^{r/a}(t-t') = \hat{g}_{ss;14}^{r/a}(t-t') = \hat{g}_{ss;23}^{r/a}(t-t') = \hat{g}_{ss;24}^{r/a}(t-t') = 0, \quad (90)$$

$$\hat{g}_{ss;31}^{r/a}(t-t') = \hat{g}_{ss;32}^{r/a}(t-t') = \hat{g}_{ss;41}^{r/a}(t-t') = \hat{g}_{ss;42}^{r/a}(t-t') = 0. \quad (91)$$

Hence the retarded(advanced) self-energy matrix $\hat{\Sigma}_R^{r/a}(\omega)$ Eq. (15) is similarly obtained from the direct matrix product $\hat{\mathbf{R}}_{pn}^\dagger \hat{\mathbf{g}}_{ss}^{r/a}(t-t') \hat{\mathbf{R}}_{pm}$ after Fourier

transformation $\int d\omega e^{i\omega t} F(t)$.

-
- ¹ For a review, see *Mesoscopic Electronic Transport*, Edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schön, (Kluwer, Series E 345, 1997).
- ² C. Kittel, *Introduction to Solid State Physics* (Willy and Sons, New York, 1976).
- ³ R. Kubo, M. Toda, and N. Hasnitsume, *Nonequilibrium Statistical Mechanics* (Springer-Verlag, Berlin, 1997).
- ⁴ S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, 1995), P246-273.
- ⁵ R. Landauer, Philos. Mag. **21**, 863 (1970); M. Büttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. B **31**, 6207 (1985).
- ⁶ W. Poirier, D. Mailly, and M. Sanquer, Phys. Rev. Lett. **79**, 2105 (1997).
- ⁷ N. van der Post, E. T. Peters, I. K. Yanson, and J. M. van Ruitenbeek, Phys. Rev. Lett. **73**, 2611 (1994).
- ⁸ A. F. Morpurgo, B. J. van Wees, T. M. Klapwijk, and G. Borghs, Phys. Rev. Lett. **79**, 4010 (1997).
- ⁹ M. T. Tuominen, J. M. Hergenrother, T. S. Tighe, and M. Tinkham, Phys. Rev. Lett. **69**, 1997 (1992).
- ¹⁰ T. M. Eiles, John M. Martinis, and Michel H. Devoret, Phys. Rev. Lett. **70**, 1862 (1993). Shashi K.
- ¹¹ S. K. Upadhyay, A. Palanisami, R. N. Louie, and R. A. Buhrman, Phys. Rev. Lett. **81**, 3247 (1999).
- ¹² M. D. Lawrence and N. Giordano, J. Phys. Condens. Mat-
ter **39**, L563(1996).
- ¹³ S. Gueron, Mandar M. Deshmukh, E. B. Myers, and D. C. Ralph, Phys. Rev. Lett. **83**, 4148 (1999).
- ¹⁴ A. F. Andreev, Sov. Phys. JETP **19**, 1228 (1964).
- ¹⁵ G. A. Prinz, Science **282**, 1660 (1998).
- ¹⁶ M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. Lett. **74**, 1657 (1999).
- ¹⁷ H. Haug and A. -P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer-Verlag, Berlin, 1998).
- ¹⁸ Y. Meir and N. S. Wingreen, Phys. Rev. Lett. **68**, 2512 (1992).
- ¹⁹ Qing-feng Sun, Jian Wang, and Tsung-han Lin, Phys. Rev. B **59**, 3831 (1999).
- ²⁰ A. Levy Yeyati, J. C. Cuevas, A. LpezDvalos, and A. MartnRodero, Phys. Rev. B **55**, R6137 (1997).
- ²¹ J. C. Slonczewski, Phys. Rev. B **39**, 6995 (1989).
- ²² P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).
- ²³ A. P. Jauho and N. S. Wingreen, Phys. Rev. B **50**, 5528 (1994).
- ²⁴ N. N. Bogoliubov, Nuovo Cimento **7**, 794 (1958); J. G. Valatin, Nuovo Cimento **7**, 843 (1958).